Functions

Assume A and B are sets. Function f from A to B is an assignment of exactly one element of B to each element of A.

$f : A \to B$

domain (A) co-domain (B)

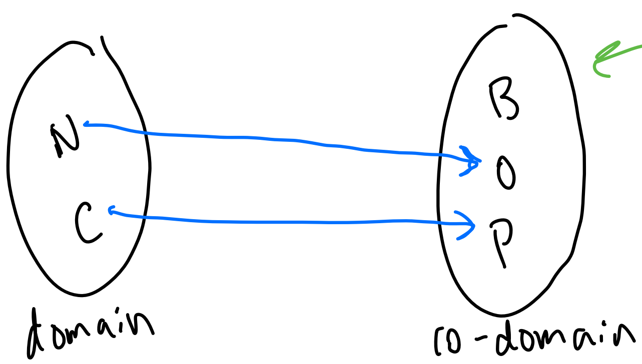
if $x \in A$, then $f(x)$ is the image of x.

ex) $A = \{\text{Naina}, \text{Charlotte}\}$

$B = \{\text{blue}, \text{orange}, \text{purple}\}$

$c : A \to B$

$c(\text{Naina}) = \text{orange}$ $c(\text{Charlotte}) = \text{purple}$



co-domain = {blue, orange, purple)

image = { orange, purple }

NOT onto

What is not a function?

$C(\text{Naina}) = \text{orange}, \; C(\text{Naina}) = \text{blue}$ NOT a function

$C(\text{Naina}) = \text{undefined}$ NOT a function

these violate "exactly one" element of B

The image of a function $f: A \to B$ is the set of values produced when f is applied to elements of A.

↳ outputs we actually get

you determine image $f: A \to B$

A function is $\boxed{\text{onto}}$ iff the image equals the $\boxed{\text{co-domain}}$.

or, $\forall y \in B, \exists x \in A$ such that $f(x) = y$

for all element in co-domain

we can find element in domain

that gives us y as an output

note about nested quantifiers: ORDER MATTERS

$\exists x \in A, \forall y \in B \quad f(x) = y$

we can find an element in the domain

that element outputs every element in the co-domain

onto proof:

Define function $g: \mathbb{Z} \to \mathbb{Z}$, $g(x) = x + 2$. Prove g is onto.

We must show that for any arbitrary element y in co-domain, it has a pre-image in the domain. In other words, $a \in \mathbb{Z}$ such that $g(a) = y$.

on side

$y = a + 2$

$a = y - 2$

(inverse)

So let $y \in \mathbb{Z}$. Then, let $a = y - 2$. We must show that $a \in \mathbb{Z}$ (domain). a must be an integer because $y, 2 \in \mathbb{Z}$.

$g(a) = a + 2 = y - 2 + 2 = y$. So we have $g(a) = y$.

So we found a pre-image for our arbitrarily chosen y, so g is onto.

function composition:

$f(x) = 3x + 7$

$g(x) = x - 8$

$(f \circ g)(x) = f(g(x))$

$f(x-8) = 3(x-8) + 7 = 3x - 24 + 7 = 3x - 17$

☆ see book for function composition proofs ☆